EINDHOVEN UNIVERSITY OF TECHNOLOGY

Data-driven model learning in interconnected systems

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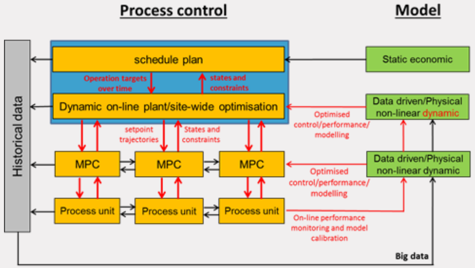
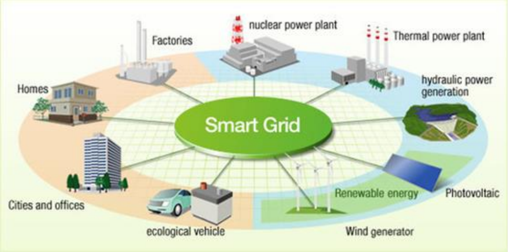
European Research Council

Introduction – dynamic networks

Decentralized process control



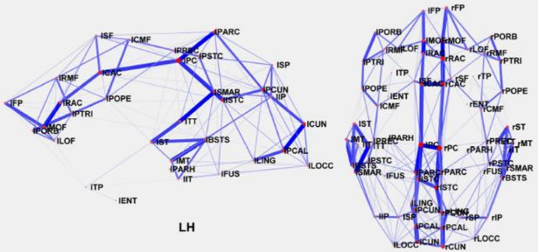
Smart power grid



Pierre et al. (2012)

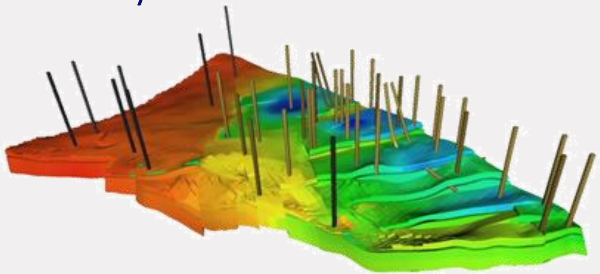
Complex machines

Brain network



P. Hagmann et al. (2008)

Hydrocarbon reservoirs



Mansoori (2014)

Introduction

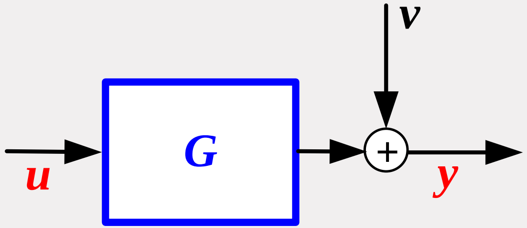
Overall trend:

- (Large-scale) interconnected dynamic systems
- Distributed / multi-agent type monitoring, control and optimization problems, as well as diagnostics
- Data is “everywhere”, big data era, AI/machine learning tools
- Model-based operations require accurate/relevant models
- → **Learning models/actions from data** (including physical insights when available)

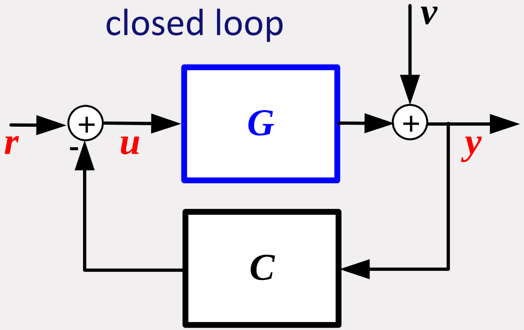
Introduction

The classical (multivariable) data-driven modeling problems[1] :

open loop



closed loop



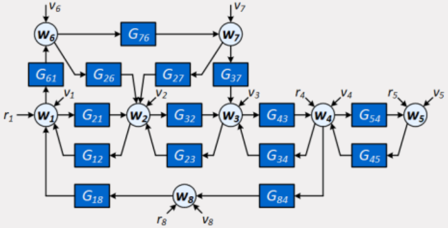
Identify a model of G on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

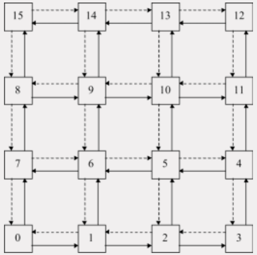
> In interconnected systems (networks) the **structure / topology** becomes important to include

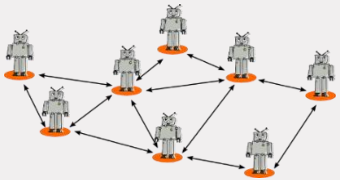
[1] Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

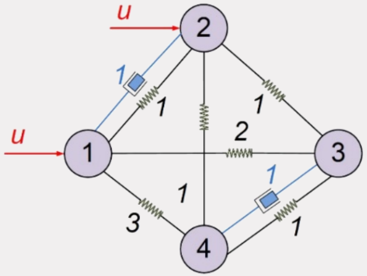
Network models

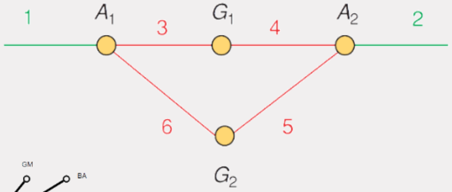
- scalable, describing the physics
- dynamic elements with cause-effect
- handling feedback loops (cycles)
- combining physical and cyber components
- centered around measured signals
- allow disturbances and probing signals

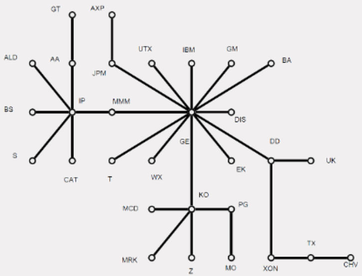


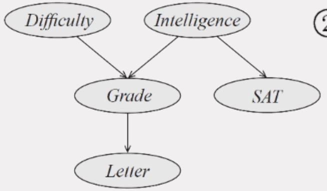


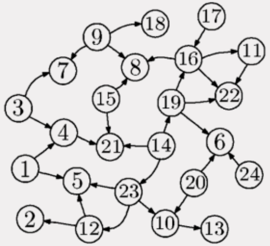


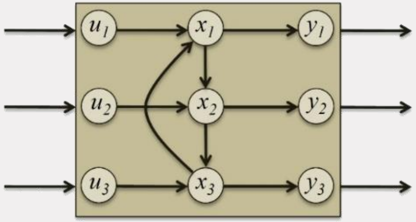


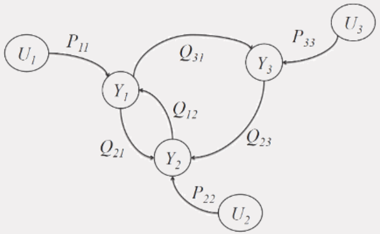






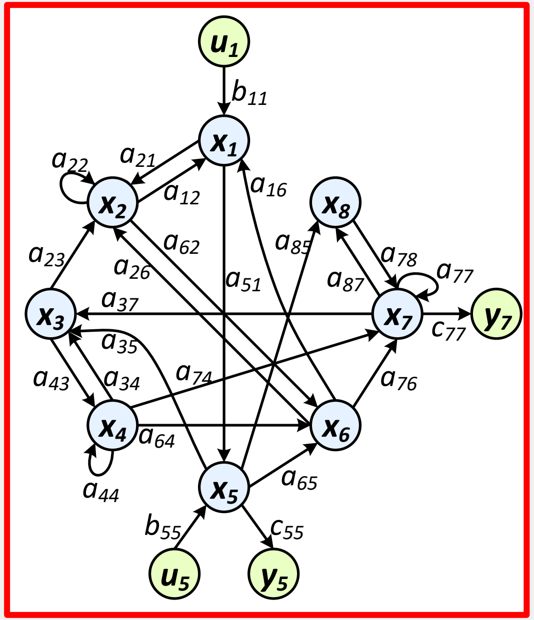






D. Materassi and M.V. Salapaka (2012)
www.momo.cs.okayama-u.ac.jp
E.A. Carara and F.G. Moraes (2008)
P.M.J. Van den Hof et al (2013)
J.C. Willems (2007)
X.Cheng (2019)
R.N. Mantegna (1999)
D. Koller and N. Friedman (2009)
P.E. Paré et al (2013)
E. Yeung et al (2010)

Network models



State space representation

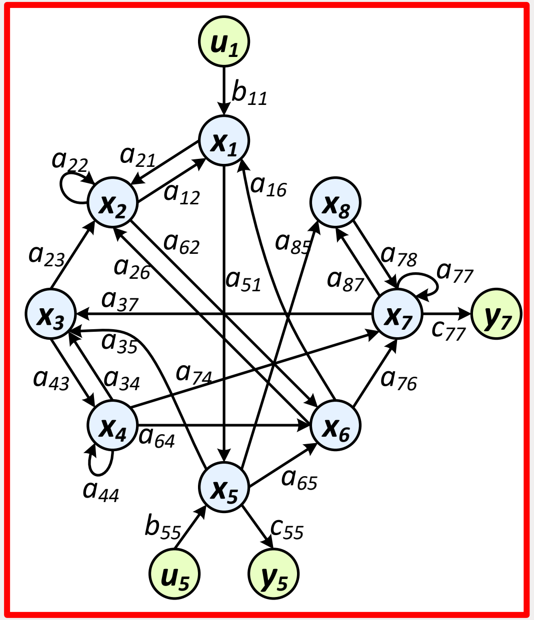
$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k) + Du(k)
\end{aligned}
$$

- States as **nodes** in a (directed) graph
- State transitions (1 step in time) reflected by a_{ij}
- Transitions are encoded in links
- Ultimate break-down of system structure
- Actuation (u) and sensing (y) reflected by separate links

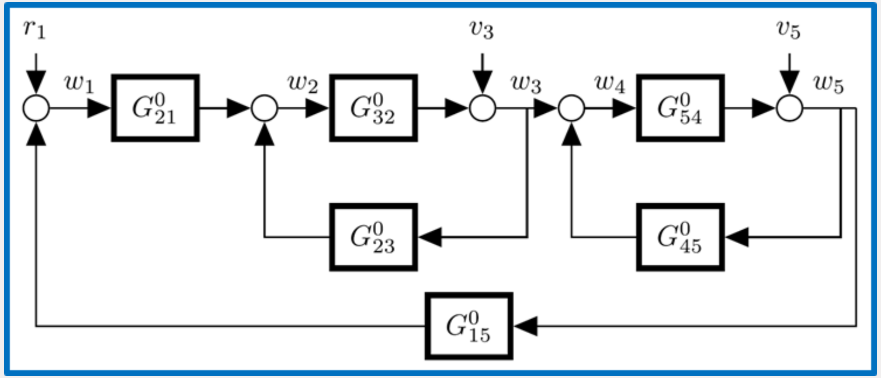
For data-driven modeling problems:

- Lump unmeasured states in dynamic **modules**

Network models



State space representation [1]

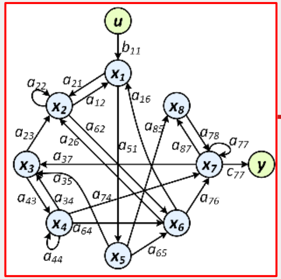


Module representation [2]

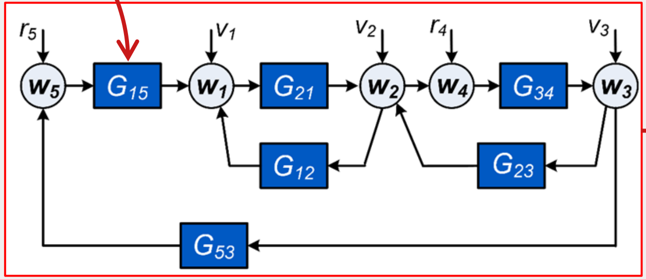
[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,…

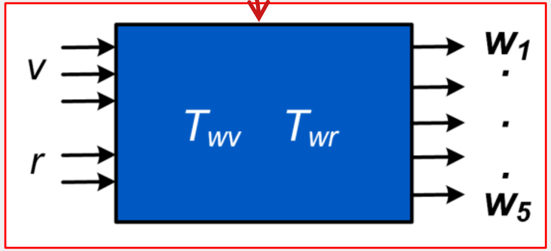
[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts,…

Dynamic network models - zooming



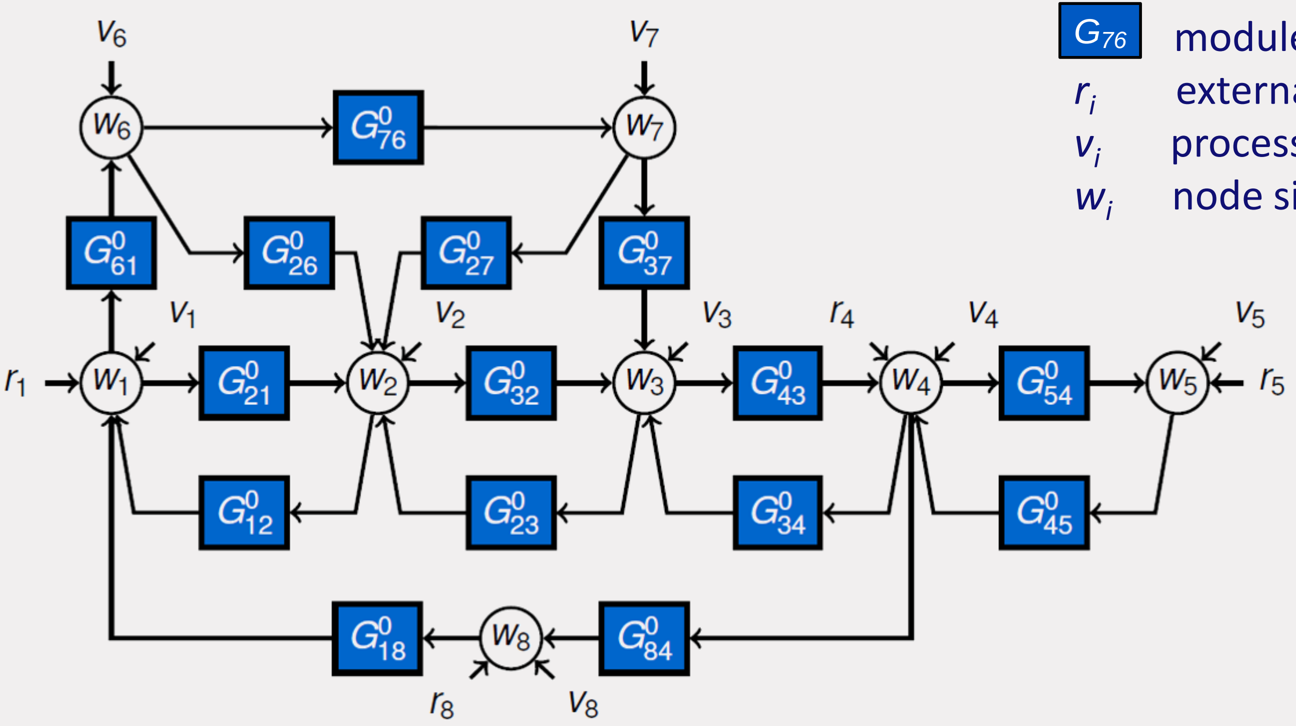
Increasing level of detail





Decreasing structural information

Dynamic network setup



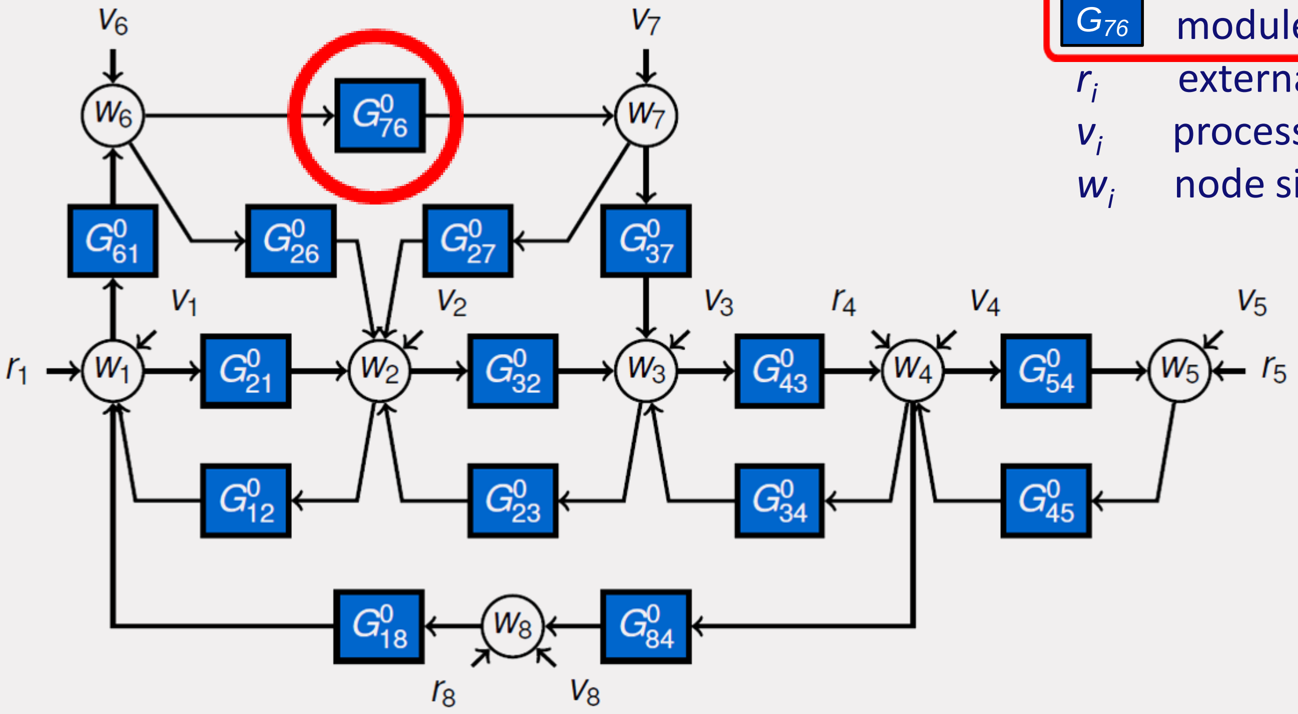
G_{76} module

r_i external excitation

v_i process noise

w_i node signal

Dynamic network setup



G_{76} module

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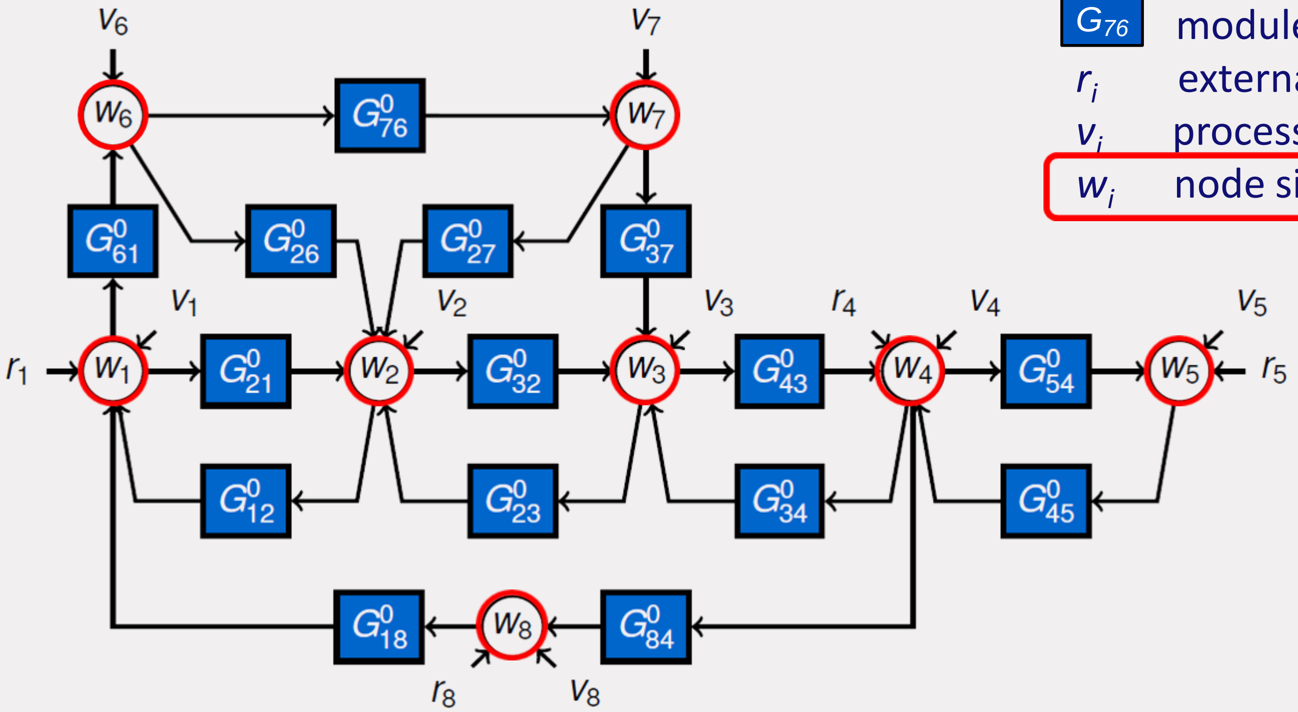
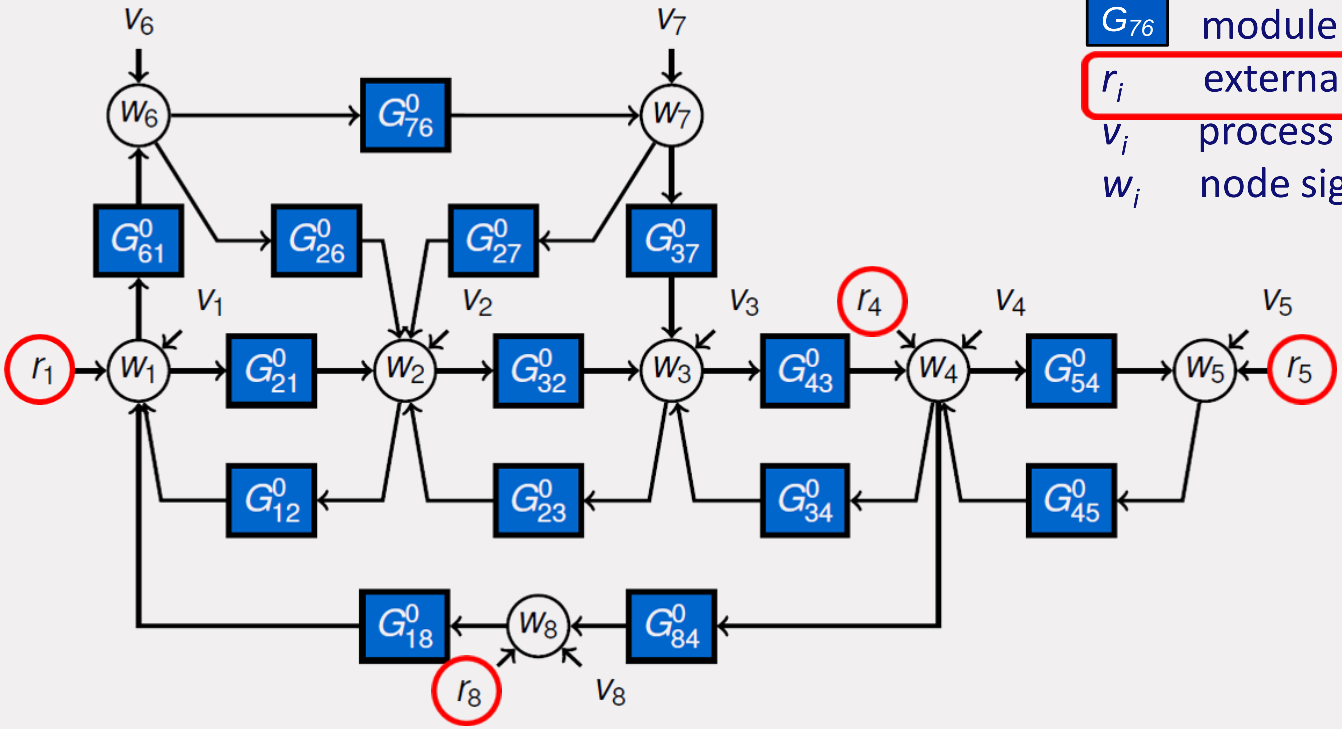
Dynamic network setup



G_{76} module
r_i external excitation
v_i process noise
w_i node signal

Dynamic network setup



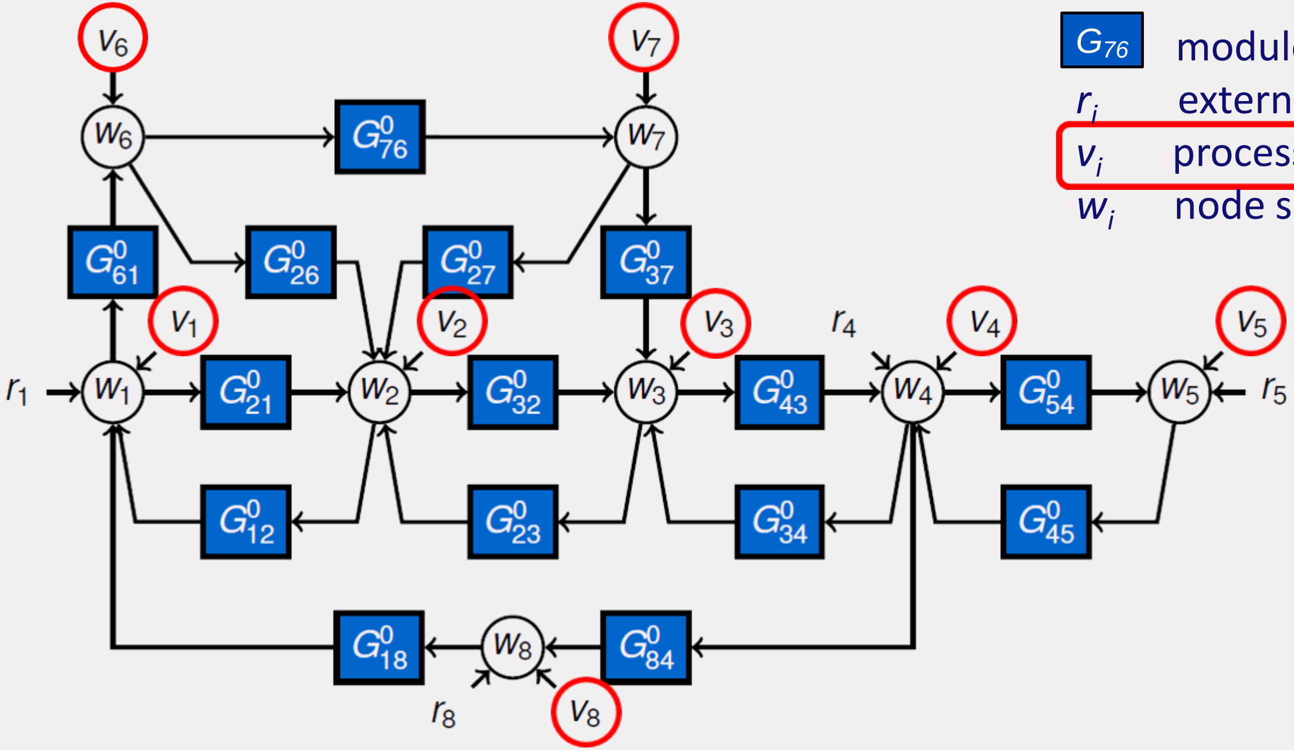
G_{76} module

r_i external excitation

v_i process noise

w_i node signal

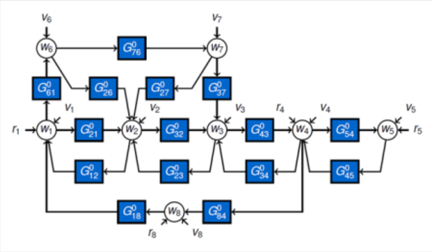
Dynamic network setup



G_{76}	module
r_i	external excitation
v_i	process noise
w_i	node signal

Dynamic network setup

Basic building block:



$$w_j(t) = \sum_{k \in \mathcal{N}_j} G^0_{jk}(q) w_k(t) + r_j(t) + v_j(t)$$

w_j: node signal
r_j : external excitation signal
v_j : (unmeasured) disturbance, stationary stochastic process
G^0_{jk}: module, rational proper transfer function, $\mathcal{N}_j \subset \{\mathbb{Z} \cap [1, L] \backslash \{j\}\}$
q: shift operator, $q^{-1}w(t) = w(t-1)$

Node signals: $w_1, \cdots w_L$
Interconnection structure / topology of the network is encoded in $\mathcal{N}_j,\ j = 1, \cdots L$

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

TU/e

Dynamic network setup

Collecting all equations:

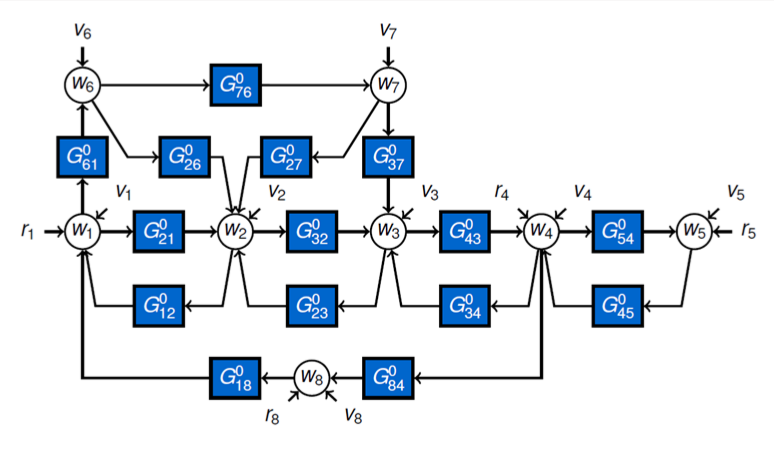
$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + H^0 \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_p \end{bmatrix}$$

$$w(t) = G^0(q)w(t) + \underbrace{R^0(q)r(t)}_{u(t)} + v(t); \qquad v(t) = H^0(q)e(t); \quad cov(e) = \Lambda$$

- Typically R^0 is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0.
- r and e are called external signals.

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

Dynamic network setup



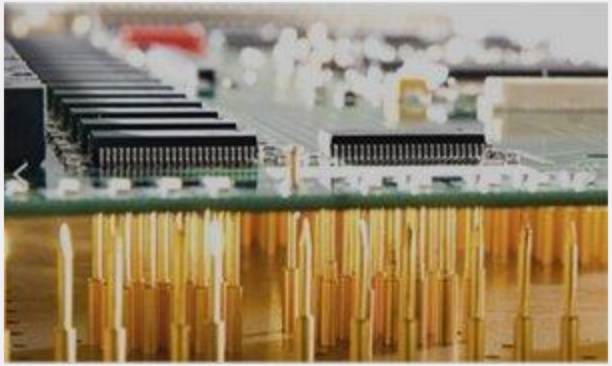
Measured time series:

$$\{w_i(t)\}_{i=1,\cdots L}; \ \{r_j(t)\}_{j=1,\cdots K}$$

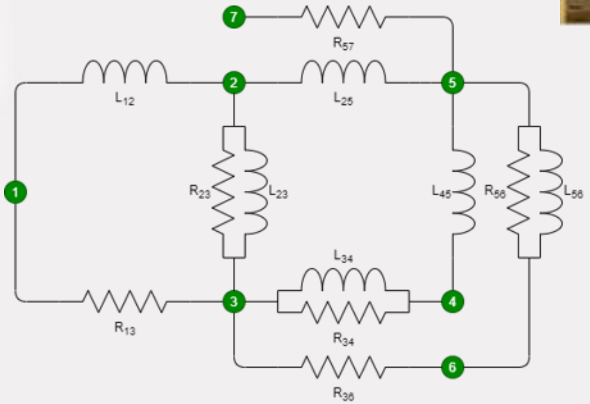
Many challenging data-driven modeling and diagnostics challenges appear

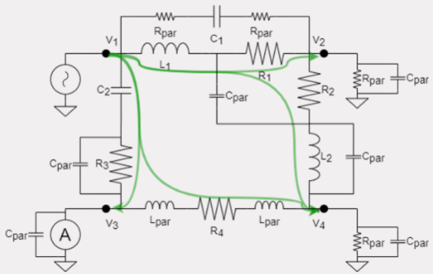
- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Diagnostics and fault detection
- User prior knowledge of modules
- Distributed identification
- Scalable algorithms

Application: Printed Circuit Board (PCB) Testing



Source: Altium



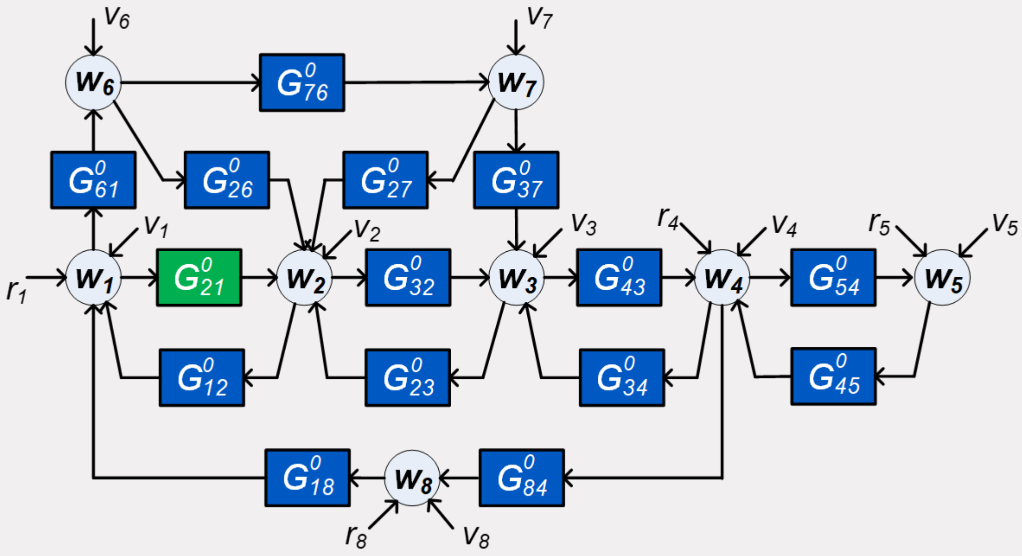


Detection of

- component failures
- parasitic effects

Single module identification

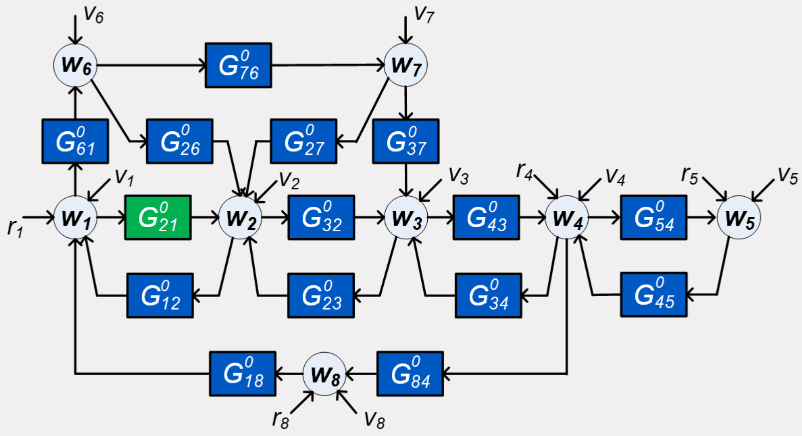
Single module identification



For a network with
known topology:

- Identify G^0_{21} on the basis of measured signals
- Which signals to measure? Preference for local measurements
- When is there enough excitation / data informativity?

Single module identification



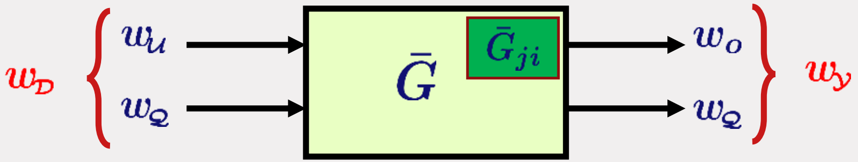
Local direct method:

(consistency and minimum variance properties)

Select a subnetwork:

- Predicted outputs: $w_{\mathcal{Y}}$
- Predictor inputs: $w_{\mathcal{D}}$

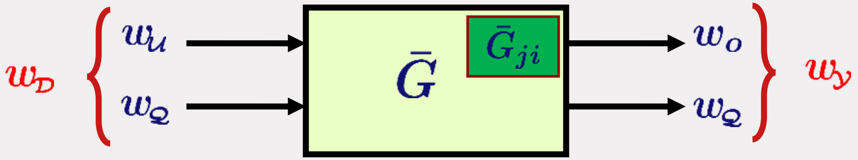
such that prediction error minimization leads to an accurate estimate of G^0_{21}



Note: same node signals can appear in input and output

TU/e

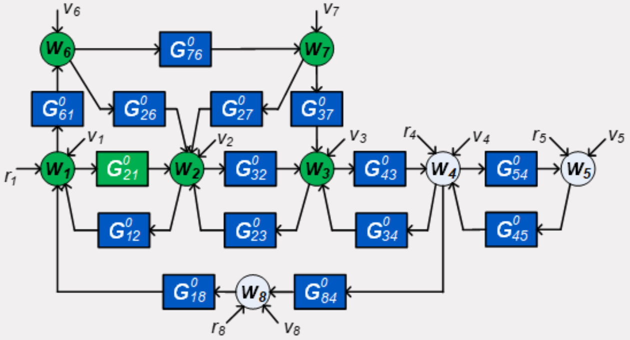
Single module identification

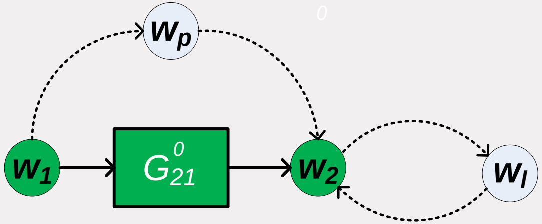


Conditions for arriving at an accurate model:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$
2. Handling of confounding variables
3. Data-informativity
4. *Technical condition on presence of delays*

Single module identification - module invariance





A sufficient condition for module invariance:

All parallel paths, and loops around the output,
should be ”blocked” by a measured node that is present in $w_{\mathcal{D}}$

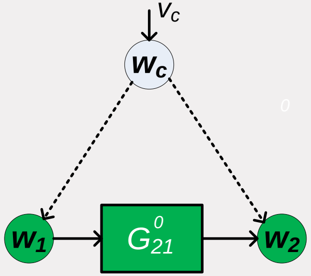
All other signals can be removed/immersed from the network

[1] Dankers et al., TAC 2016
[2] Shi et al., Automatica 2022

Single module identification – confounding variables

Confounding variable [1][2]**:**

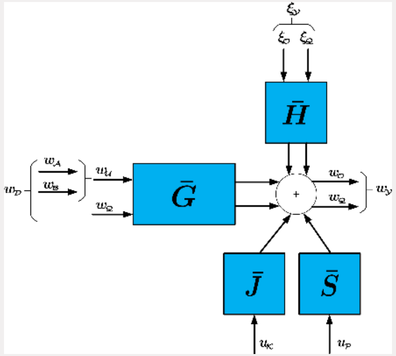
Unmeasured signal that has (unmeasured paths) to both the input and output of an estimation problem.



In networks they can appear in two different ways:

- If v disturbances on inputs and outputs are correlated
- If non-measured in-neighbors of $w_{\mathcal{Y}}$ affect signals in $w_{\mathcal{D}}$

Can be addressed by **adding inputs/outputs** to the predictor model[3]



[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009
[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.
[3] K.R. Ramaswamy et al., IEEE-TAC, 2021.

Single module identification – data-informativity

Predictor model equation:

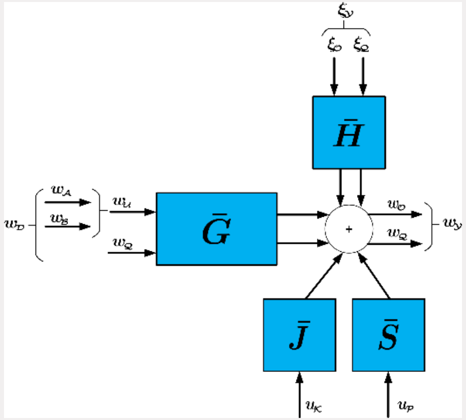
$$w_{\mathcal{Y}}(t) = \bar{G}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t)$$

Typical data-informativity condition:

κ persistently exciting $\quad \boxed{\Phi_\kappa(\omega) > 0 \text{ for almost all } \omega}$

$$\kappa(t) := \begin{bmatrix} w_{\mathcal{D}}(t) \\ \xi_{\mathcal{Y}}(t) \\ u_{\mathcal{K}}(t) \end{bmatrix}$$

inputs of the predictor model



Rank-based condition can generically be satisfied based on a graph-based condition

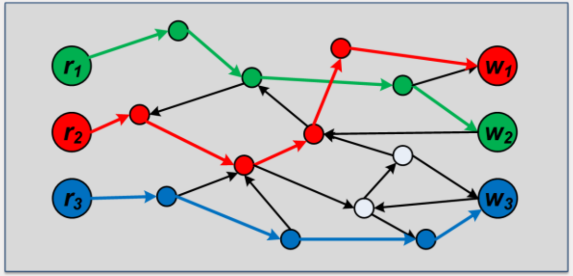
[1] K.R. Ramaswamy et al., IEEE-TAC, 2021.

TU/e

Data informativity (path-based condition)

This condition can be verified in a generic sense, by considering the **generic rank** of the mapping from external signals to κ [1],[2]

linking to the maximum number of **vertex disjoint paths** between inputs and outputs



$b_{\mathcal{R}\to\mathcal{W}} = 3$

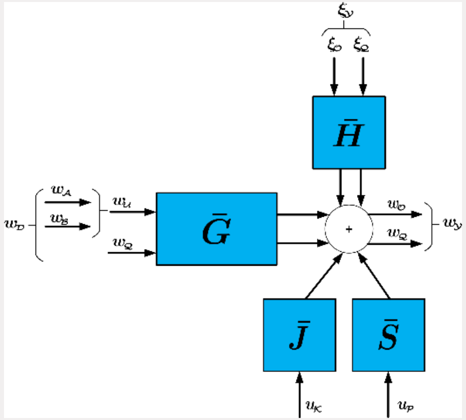
κ persistently exciting holds **generically** if there are **vertex disjoint paths** between external signals $\{u, e\}$and $\kappa = \begin{bmatrix} w_{\mathcal{D}} \\ \xi_{\mathcal{Y}} \\ u_{\mathcal{K}} \end{bmatrix}$

Equivalently:

$dim(w_{\mathcal{D}})$ vertex disjoint paths between $\{u, e\} \backslash \{\xi_{\mathcal{Y}}, u_{\mathcal{K}}\}$ and $w_{\mathcal{D}}$

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019.
[3] VdH et al., CDC 2020.

Data informativity (path-based condition)



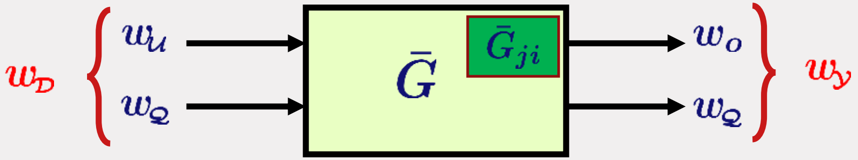
Specific result for networks with **full rank disturbances**:

Every node signal in $w_{\mathcal{Q}}$ requires an excitation in $u_{\mathcal{P}}$ having a 1-transfer to $w_{\mathcal{Y}}$

$$w_{\mathcal{Y}}(t) = \bar{G}(q,\theta)w_{\mathcal{D}}(t) + \bar{H}(q,\theta)\xi_{\mathcal{Y}}(t) + \bar{J}(q,\theta)u_{\mathcal{K}}(t) + \bar{S}u_{\mathcal{P}}(t)$$

- For every node in $w_{\mathcal{Q}}$ we need a *u*-excitation
- More expensive experiments with growing # outputs

Single module identification



Conditions for arriving at an accurate model:

1. Module invariance: $\bar{G}_{ji} = G^0_{ji}$
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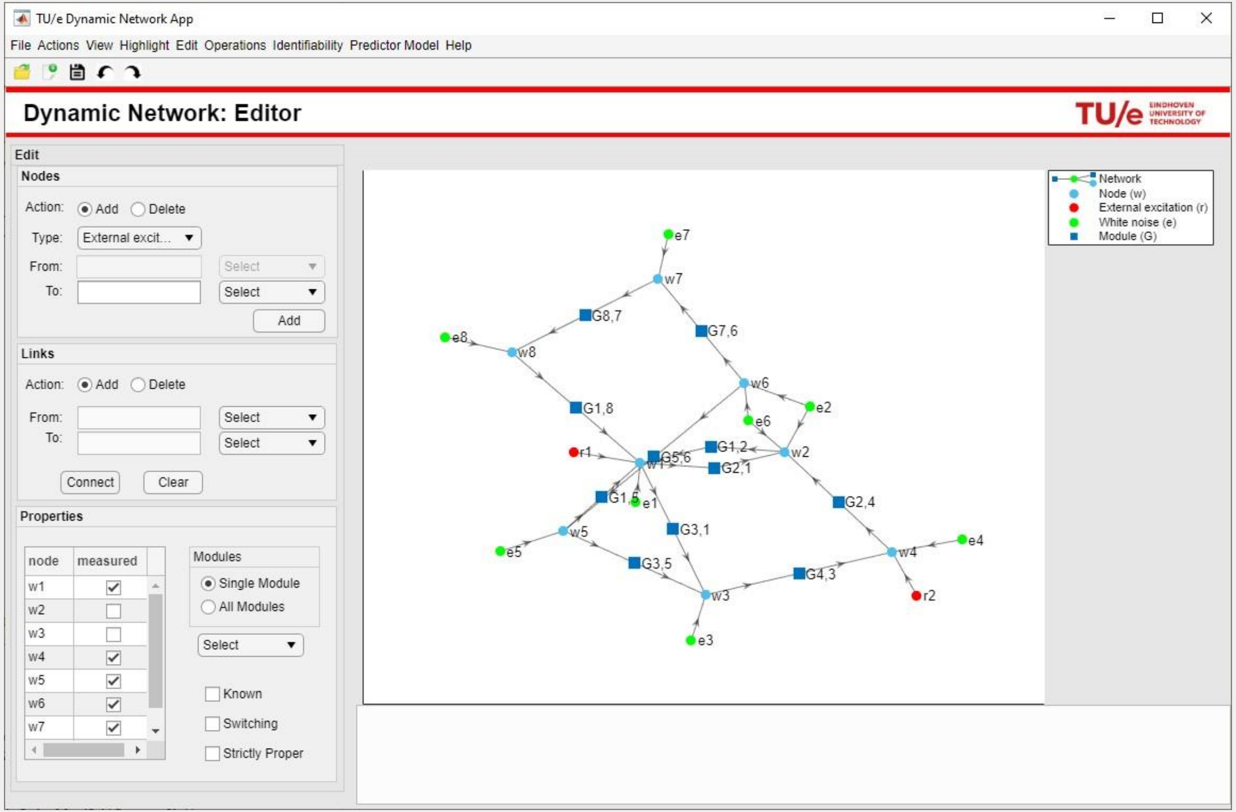
Path-based conditions on the network graph

Algorithms implemented in SYSDYNET Toolbox

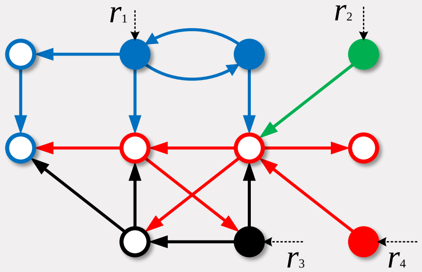


Summary single module identification

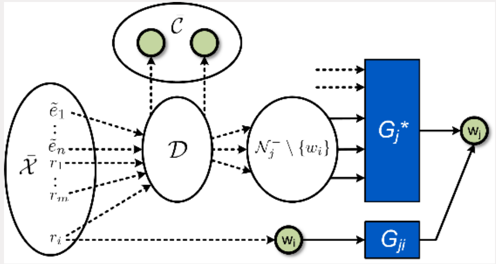
- Path-based conditions that the predictor model should satisfy
- Different algorithms for synthesizing predictor model
- Degrees of freedom in sensor / actuator placement
- Methods for **consistent** and **minimum variance** module estimation, and effective (scalable) algorithms

Related topics...

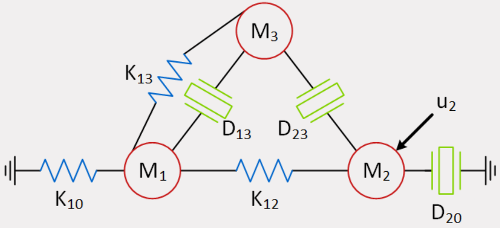
- Excitation allocation for full network identifiability



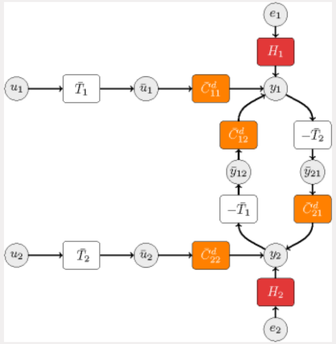
- Subnetwork identifiability



- Diffusively coupled networks



- Distributed controller identification



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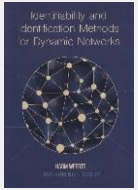
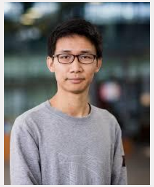
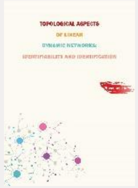


Harm Weerts

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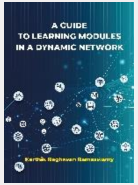

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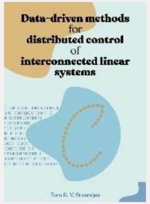

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Further reading

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